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ALLOCATION OF ADVERTISING AND RESEARCH DOLLARS IN THE FLORIDA ORANGE JUICE INDUSTRY: SUPPLY AND DEMAND ELASTICITY CONSIDERATIONS

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Allocation of Advertising and Research Dollars In the Florida Orange-Juice Industry: Supply and Demand Elasticity Considerations

The Florida citrus industry spends money on both production research and advertising. Much of the money available for these activities comes from self-imposed taxes on Florida citrus growers. The largest tax is collected by the Florida Department of Citrus (FDOC) and the largest portion of this money has been used for promotion and advertising. The FDOC also funds post-harvest research, as well as research on mechanical harvesting. Growers also pay a tax specifically for production research. Additionally, State-of-Florida and Federal monies are used to support citrus research through the University of Florida and the USDA.

The advertising by the FDOC focuses on increasing the demand for Florida citrus products, while the State's production research focuses on increasing supply (supplying more at a given price or supplying the same at a lower price). Florida's major citrus product, orange juice (OJ), is advertised nationally through TV commercials and other media. The FDOC post-harvest research has resulted in various improvements in the processing and fresh packaging sectors, including, for example, the discovery of frozen concentrated orange juice (FCOJ) in the early 1940s. Production oriented research conducted by the University of Florida and the USDA has resulted in new fruit varieties, new technologies and production practices.

With tens of millions of dollars spent on advertising and research, a question is "what is the best allocation of a given budget to these two alternatives?" This has become a particularly important question given the recent threat that two diseases, citrus canker and greening, pose to Florida citrus production. There must be production, of course, to have an industry, which will require a major research effort to fight these diseases, but, on the other hand, there must also be sufficient demand for Florida citrus growers to earn a return that keeps them in business.

Various studies have found that FDOC advertising has had substantial impacts on OJ demand (e.g., FABA; Ward et al; MAP; Brown; and Brown and Lee). A study on post-harvest research in the Florida citrus processing sector also found that this activity had a high rate of return (Stranahan; and Shonkwiler and Stranahan). Studies on the gains from research and promotion for other commodities have also been conducted (e.g., Wohlgenant; Chung and Kaiser; Chyc and Goddard; Cranfield; and Fuglie and Heisley). In this paper, some mathematical expressions are developed to address the advertising-research allocation issue. The focus is on maximization of grower revenue and the role of supply and demand elasticities. A simple graphical analysis on the relationship between revenue, and supply and demand price elasticities is first provided. Several simple advertising/research-impact models are then presented to further introduce basic concepts, followed by a more generalized optimal allocation result in context of a world model for OJ. An application of the optimality conditions to examine the advertising-research mix for the Florida citrus industry is then discussed.

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Revenue and Supply and Demand Price Elasticities

The elasticities of supply and demand with respect to price, along with the magnitudes of the supply and demand shifts due to research and advertising, respectively, are basic factors that determine revenue. For a linear relationship between quantity demanded and price, the elasticity of demand is greater than unity in absolute value or demand is in the elastic range at relatively high prices (p1 and p2 in Figure 1). In this figure, suppose, demand is initially represented by D1, supply is perfectly inelastic as indicated by S1, and price is p1. Associated revenue is given by the rectangle enclosing the area p1 times q1. Now if, as a result of advertising, demand increases to D2, price will increase to p2, and revenue will increase by the amount (p2-p1) times q1, as shown by the associated upward arrow. Alternatively, suppose that supply is perfectly elastic at price p1. In this case, the increase in demand would not result in a change in price, but sales would increase to q2, and revenue would increase by the amount (q2-q1) times p1, as shown by the associated arrow pointing to the right. Given demand is elastic, the revenue gain for the latter elastic supply case would exceed that for the former case where supply is inelastic. Thus, if demand is elastic, advertising would result in a greater revenue gain if supply were perfectly elastic, as opposed to perfectly inelastic. A similar conclusion is reached if we assume demand is unchanged at D2 and let supply be perfectly inelastic, increasing, say as a result of research, from S1 (q1) to the vertical line associated with q2. In this case, sales increase by the amount q2-q1, while price decreases by the amount p2-p1. The gain in revenue, p1 times (q2-q1) exceeds the loss, (p2-p1) times q1, and thus research expands revenue when demand is elastic.

Opposite results occur at points where demand is inelastic. At relatively low prices (p3 and p4), the elasticity of demand is less than unity in absolute value or demand is in the inelastic range. Again suppose demand is initially D1, but let supply be perfectly inelastic as indicated by S3. In this case, price is p3, and associated revenue is given by the rectangle enclosing the area p3 times q3. For an advertising-induced increase in demand to D2, price will increase to p4, and revenue will increase by the amount (p4-p3) times q3, as shown by the associated upward arrow. On the other hand, if supply were perfectly elastic at price p3, there would be no price change for the increase in demand, but sales would increase to q4, and revenue would increase by the amount (q4-q3) times p3, as shown by the associated arrow pointing to the right. Given demand is inelastic, the revenue gain for the inelastic supply case would be more than that for the elastic supply situation. Thus, when demand is inelastic, advertising would result in a greater revenue gain if supply were perfectly inelastic, as opposed to being perfectly elastic. Similarly, if demand is unchanged at D2 and supply is perfectly inelastic, but, as a result of research, increases from S3 (q3) to the vertical line associated with q4, price declines from p4 to p3, and the revenue declines by the amount (p4-p3) time q3, and increases by the amount p3 times (q4-q3), and since the revenue loss exceeds the gain, research is not effective at expanding revenue when demand is inelastic.

From these simple illustrations it thus appears that the elasticities of demand and supply are integral parts of the revenue maximization problem for advertising and research.

Optimal Advertising, Price Constant
(Perfectly Elastic or Unlimited Supply at the Consumer Price)

Initially, assume that the demand for OJ demand can be represented by the double log equation

(1) $q = \alpha p^{\varepsilon p} A^{\varepsilon a}$,

where q is quantity, p is price, A is advertising dollars, ε_p and ε_a are price and advertising elasticities ($\varepsilon_p < 0$; $\varepsilon_a > 0$), respectively, and α represents the effects of all other factors.

The associated total revenue (R) is

(2) $R = pq = p\alpha p^{\varepsilon p} A^{\varepsilon a}$

With the total cost of advertising being A, revenue net of advertising or profit (total revenue-total cost) is then

(3a) $\pi = pq - A$

or

(3b) $\pi = p\ \alpha p^{\varepsilon p} A^{\varepsilon a} - A.$

The first order condition for maximizing net revenue (3a) and (3b) is

(4a) $\partial\pi/\partial A = \partial pq/\partial A - 1 = 0$

or

(4b) $\partial\pi/\partial A = \varepsilon_a\, pq / A - 1 = 0.$

Marginal revenue(MR) with respect to advertising is $\partial R/\partial A = \varepsilon_a\, pq / A$, while marginal cost of advertising (MC) is 1 ($MC = \partial A/\partial A = 1$). Hence from equation (4b), the optimal condition is MR = MC, i.e.,

(5a) $\varepsilon_a\, pq / A = 1,$

or

(5b) $A = \varepsilon_a pq.$

Equation (5b) indicates that to maximize revenue, price constant, advertising equals the elasticity of demand with respect to advertising times total revenue, where q is set at the optimal level.

Advertising A appears on both the left and right hand sides of equation (5b). On the right hand side, quantity demanded q is a function of advertising. Thus substituting the right and side of equation (1) for q in equation (5b) results in

(5c) $A = \varepsilon_a p\ \alpha p^{\varepsilon p} A^{\varepsilon a}$,

or, solving for A,

(5d) $A = (\varepsilon_a p\ \alpha p^{\varepsilon p})^{1/(1-\varepsilon a)}$.

The second order condition is

(6a) $\partial^2\pi/\partial A\ \partial A = \varepsilon_a\ \varepsilon_a\ pq/A^2 - \varepsilon_a\ pq/A^2 < 0,$

or

(6b) $(pq/A^2)\ \varepsilon_a(\varepsilon_a - 1) < 0,$

which is satisfied when

(6c) $0 < \varepsilon_a < 1.$

An example is used to illustrate the optimal level of advertising under the above conditions. Consider two periods, t-1 and t, where price is the same across periods but advertising in period t-1 is non-optimal. The problem is to determine the optimal level of advertising in period t. Letting quantity, price and advertising in period t-1 be predetermined, from equation (1), $\alpha = q_{t-1} / (p_{t-1}^{\varepsilon p} A_{t-1}^{\varepsilon a})$, and substituting this result into (5d) evaluated at time t, $A_t = (\varepsilon_a p_t (q_{t-1} / (p_{t-1}^{\varepsilon p} A_{t-1}^{\varepsilon a}))\ p_t^{\varepsilon p})^{1/(1-\varepsilon a)}$, or given price is the same in each period, $A_t = (\varepsilon_a p_t\ q_{t-1} / A_{t-1}^{\varepsilon a})^{1/(1-\varepsilon a)}$. Letting $\varepsilon_a = .07$, $p_t = p_{t-1} = \$1.40$ per pound solid (ps) and $q_{t-1} = 1200$ million ps and $A_{t-1} = \$30$ million, the optimal advertising in period t is $A_t = (.07{*}1.40{*}1200/30^{.07})^{1/.93} = \$130.3$ million.

Optimal Advertising, Quantity Constant
(Perfectly Inelastic or Fixed Supply)

Inverting the direct demand $q = \alpha p^{\varepsilon p} A^{\varepsilon a}$, the price-dependent double log OJ demand is

(7) $p = (q\ A^{-\varepsilon a}/\alpha)^{1/\varepsilon p}$

The corresponding revenue is

(8) $R = pq = q(q\,A^{-\varepsilon a}/\alpha)^{1/\varepsilon p}$,

and the corresponding marginal revenue with respect to advertising is

(9) $MR = -(\varepsilon_a/\varepsilon_p)\,pq/A$.

Thus the optimal condition (MR = MC) is

(10a) $-(\varepsilon_a/\varepsilon_p)\,pq/A = 1$

or

(10b) $A = -(\varepsilon_a/\varepsilon_p)\,pq$,

or, substituting the right hand side of equation (8) for pq,

(10c) $A = -(\varepsilon_a/\varepsilon_p)\,q\,(q\,A^{-\varepsilon a}/\alpha)^{1/\varepsilon p}$,

or, solving for A,

(10c) $A = -((\varepsilon_a/\varepsilon_p)\,q\,(q/\alpha)^{1/\varepsilon p})^{(1/(1+\varepsilon a/\varepsilon p))}$.

Equation (10b) indicates that to maximize revenue, quantity fixed, advertising equals the elasticity of demand with respect to advertising times total revenue divided by the absolute value of the price elasticity of demand, where q is again set at the optimal level.

Assuming the same solution for α in the previous example, the quantity in both periods is the same ($q_t=q_{t-1}$) at 1200 million ps, and the elasticity of demand is -.5, the optimal level of advertising in period t is \$328.9 million.

Optimal Advertising, Quantity and Price Endogenous

In addition to the previous demand equation, $q = \alpha p^{\varepsilon p} A^{\varepsilon a}$, assume that supply is

(11) $q = \gamma\, p^{\eta p}$,

where η_p is the elasticity of quantity supplied with respect to price ($\eta_p \geq 0$).

Price and quantity are endogenous while advertising is exogenous. The reduced form equation for p, which expresses price as a function of advertising, can be found by equating supply and demand and solving for p, i.e., (12a) $\gamma\, p^{\eta p} = \alpha p^{\varepsilon p} A^{\varepsilon a}$,

or

(12b) $p^{\eta p - \varepsilon p} = (\alpha/\gamma) A^{\varepsilon a}$,

or

(12c) $p = (\alpha/\gamma)^{1/(\eta p - \varepsilon p)} A^{\varepsilon a/(\eta p - \varepsilon p)}$.

The reduced form equation for q can be found by substituting the right-hand side of equation (12c) into equation (11) for p, i.e.,

(13) $q = \gamma\, (\alpha/\gamma)^{(\eta p/(\eta p - \varepsilon p))} A^{(\eta p \varepsilon a/(\eta p - \varepsilon p))}$

Substituting the right-hand sides of the reduced form equations for price and quantity, equations (12c) and (13), into equation (3a), and differentiating with respect to advertising, the optimality condition is now

(14a) $\partial\pi/\partial A = \partial pq/\partial A - 1 = 0$

or

(14b) $\partial\pi/\partial A = q\, \partial p/\partial A + p\, \partial q/\partial A - 1 = 0$

or, after multiplying through by A, and multiplying and dividing selected terms by p and q,

(14c) $(\partial\pi/\partial A)A = pq\, (\partial p/\partial A)A/p + pq\, (\partial q/\partial A)A/q - A = 0$

or

(14d) $A = pq(\, \varepsilon_a/(\eta_p - \varepsilon_p) + \eta_p\, \varepsilon_a/(\eta_p - \varepsilon_p)\,)$

or

(14e) $A = (\, pq\, \varepsilon_a/(\eta_p - \varepsilon_p)\,)\, (1 + \eta_p)$.

or, substituting equations (12) and (13) for p and q, respectively,

(14f) $A = (\, (\alpha/\gamma)^{1/(\eta p - \varepsilon p)}\, \gamma\, (\alpha/\gamma)^{(\eta p/(\eta p - \varepsilon p))}\, \varepsilon_a/(\eta_p - \varepsilon_p)\,)\, (1 + \eta_p)\,)^{1/(1 + \varepsilon a(1 + \eta p)/(\eta p - \varepsilon p))}$

When $\eta_p = \infty$, equation (14e) is the price constant case; when $\eta_p = 0$, it is the quantity constant case. For the previous numerical assumptions on the advertising and price elasticities of demand, and base values in for period t-1, plus an assumed elasticity of supply (η_p) of .5, the optimal level of advertising is $282.9 million.

Optimal Research, Quantity and Price Endogenous

Continue to assume the above demand specification, but now let supply be extended to be a function of research expenditures R, i.e.,

(15) $q = \gamma\, p^{\eta p} R^{\varepsilon r}$,

where ε_r is the elasticity of quantity supplied with respect to research ($\varepsilon_r > 0$).

Equating demand equation (1) and supply equation (15), the reduced form equation for price is found as follows

(16a) $\gamma p^{\eta p} R^{\varepsilon r} = \alpha p^{\varepsilon p} A^{\varepsilon a}$

or

(16b) $p^{\eta p - \varepsilon p} = (\alpha/\gamma) A^{\varepsilon a} R^{-\varepsilon r}$

or

(16c) $p = (\alpha/\gamma)^{1/(\eta p - \varepsilon p)}\, A^{\varepsilon a/(\eta p - \varepsilon p)}\, R^{-\varepsilon r/(\eta p - \varepsilon p)}$

Substituting the reduced form for price (16c) into supply equation (15), the reduced form for quantity is

(17) $q = \gamma\, (\alpha/\gamma)^{(\eta p/(\eta p - \varepsilon p))}\, A^{(\eta p \varepsilon a/(\eta p - \varepsilon p))} R^{-\eta p \varepsilon r/(\eta p - \varepsilon p)}\, R^{\varepsilon r}$.

The objective function is

(18) $\pi = pq - A - R$.

Letting A be constant, then the optimality condition for research R is

(19a) $\partial\pi/\partial R = \partial pq/\partial R\ - 1 = 0$

or

(19b) $\partial\pi/\partial R = q\, \partial p/\partial R + p\, \partial q/\partial R - 1 = 0$

or, after multiplying through by R, and multiplying and dividing selected terms by p and q,

(19c) $(\partial\pi/\partial R)R = pq\,(\partial p/\partial R)R/p + pq\,(\partial q/\partial R)R/q - R = 0$

or

(19d) $R = pq(-\varepsilon_r/(\eta_p - \varepsilon_p) + -\eta_p\,\varepsilon_r/(\eta_p - \varepsilon_p) + \varepsilon_r)$

or

(19e) $R = pq(-\varepsilon_r/(\eta_p - \varepsilon_p) - \varepsilon_p\,\varepsilon_r/(\eta_p - \varepsilon_p))$

or

(19f) $R = pq\,\varepsilon_r(-\varepsilon_p - 1)/(\eta_p - \varepsilon_p)$

or

(19g) $R = ((\alpha/\gamma)^{1/(\eta p - \varepsilon p)}\,A^{\varepsilon a(1+\eta p)/(\eta p - \varepsilon p)}\,\gamma(\alpha/\gamma)^{\eta p/(\eta p - \varepsilon p)}\,\varepsilon_r(-\varepsilon_p - 1)/(\eta_p - \varepsilon_p))^{1/(1+\varepsilon r(1+\varepsilon p)/(\eta p - \varepsilon p))}$.

Result (19f) indicates that demand must be elastic ($-\varepsilon_p > 1$) for a positive R, assuming, $\eta_p > 0$, and $\varepsilon_p < 0$. Given grower demands for many agriculture commodities, including OJ, are inelastic, this result questions why there should be any production research for these commodities if the goal is to maximize revenue. This issue will be explored further in context of a world model.

The second order condition is

(20a) $\partial\pi/\partial R = -pq\,\varepsilon_r/(\eta_p - \varepsilon_p)/R - pq\,\varepsilon_p\,\varepsilon_r/(\eta_p - \varepsilon_p)/R - 1,$

or

(20b) $\partial\pi/\partial R = -(pq/R)(\varepsilon_r\,(1 + \varepsilon_p)/(\eta_p - \varepsilon_p)),$

and

(20c) $$\begin{aligned}\partial^2\pi/\partial R\,\partial R = {} & (pq/R^2)(\varepsilon_r\,(1 + \varepsilon_p)/(\eta_p - \varepsilon_p))\,(\varepsilon_r/(\eta_p - \varepsilon_p)) \\ & +(pq/R^2)(\varepsilon_r\,(1 + \varepsilon_p)/(\eta_p - \varepsilon_p))\,(\varepsilon_p\,\varepsilon_r/(\eta_p - \varepsilon_p)) \\ & +(pq/R^2)(\varepsilon_r\,(1 + \varepsilon_p)/(\eta_p - \varepsilon_p)) < 0,\end{aligned}$$

or

(20d) $(\varepsilon_r/(\eta_p - \varepsilon_p)) + (\varepsilon_p\, \varepsilon_r/(\eta_p - \varepsilon_p))+1 < 0,$

or

(20e) $-(\varepsilon_r/(\eta_p - \varepsilon_p))\,(1 + \varepsilon_p) > 1,$

or,

(20f) $\varepsilon_r < (\eta_p - \varepsilon_p)/\,(-1 - \varepsilon_p).$

Optimal Advertising and Research for a Given Budget

Assume total expenditures on advertising and research are fixed at X, i.e., X = A + R What are the optimal levels of A and R now?

The objective functions is

(21a) $\pi = pq - (A + R)$

or

(21b) $\pi = pq - X,$

where the constraint is imposed by substituting X-A for R in the reduced form equations (16c) and (17) that specify p and q, respectively, i.e.,

(22a) $p = (\alpha/\gamma)^{1/(\eta p-\varepsilon p)}\, A^{\varepsilon a/(\eta p-\varepsilon p)}\,(X\text{-}A)^{-\varepsilon r/(\eta p-\varepsilon p)}$

and

(22b) $q = \gamma\,(\alpha/\gamma)^{(\eta p/(\eta p-\varepsilon p))}\, A^{(\eta p\varepsilon a/(\eta p-\varepsilon p))}(X\text{-}A)^{-\varepsilon r\varepsilon p/(\eta p-\varepsilon p)}.$

The optimality condition is then

(23a) $\partial\pi/\partial A = q\,\partial p/\partial A + p\,\partial q/\partial A = 0$

or, multiply both sides of equation (23a) by A, and multiplying and dividing selected terms by p and q,

(23b) $(\partial\pi/\partial A)A = pq\,(\partial p/\partial A)A/p + pq\,(\partial q/\partial A)A/q = 0$

or

(23c) $(\partial p/\partial A)A/p = -(\partial q/\partial A)A/q$

or, based on equations (22a) and (22b),

(23d) $\varepsilon_a/(\eta_p-\varepsilon_p) + (A/(X-A))\ \varepsilon_r/(\eta_p-\varepsilon_p) = -\ (\eta_p\ \varepsilon_a/\ (\eta_p-\varepsilon_p)) - (A/(X-A))\ \ \varepsilon_r\ \varepsilon_p/(\eta_p-\varepsilon_p)$

or

(23e) $(A/(X-A))\ (\ \varepsilon_r(\ -\varepsilon_p\ -1)/(\eta_p-\varepsilon_p)) = (\ \varepsilon_a(1+\ \eta_p)\ /(\eta_p-\varepsilon_p)$

or

(23f) $A/R = ((\ \varepsilon_a(1+\ \eta_p)\ /(\eta_p-\varepsilon_p))\ /\ (\ \varepsilon_r\ (-\varepsilon_p\ -1)/(\eta_p-\varepsilon_p))$

or

(23g) $A/R = (\varepsilon_a/\ \varepsilon_r)\ (1+\ \eta_p)/(-\varepsilon_p\ -1)$.

Given A/R, the individual levels of A and R can be calculated from the budget constraint, i.e., letting b =A/R, then A = bR, and given X = A +R, then X = bR +R or R = X/(1+b) and A = bX /(1+b).

Result (23g) can also be obtained by dividing equation (14e), optimal advertising with research (implicitly) given, by equation (19f), optimal research with advertising given. In each of these latter two equations, the marginal impacts can be more than one dollar (given the expenditure constraint), but they must be the same.

The revenue maximizing advertising-research ratio A/R increases proportionately with the elasticity of demand with respect to advertising, and decreases with the elasticity of supply with respect to research.

Again, for result (23g), demand must be elastic ($-\varepsilon_p>1$) for a positive R. Also, in the case when demand is linear with respect to price, or for specifications where the price elasticity of demand varies from the elastic to inelastic ranges, maximum revenue would always occur when quantity sold is at the point where demand has a unity price elasticity ($-\varepsilon_p = 1$); at this point, marginal revenue is zero. Thus, if the industry is operating at a point where demand is elastic, research would be used at most to increase production to the point where demand has a unitary price elasticity. Alternatively, if production occurred at a point where demand were inelastic, no research would occur and quantity sold would be reduced until marginal revenue were zero or demand again has a unitary price elasticity.

World Model: Optimal Advertising and Research for a Given Budget

The previous result that optimal research occurs when demand is elastic is for a single market or country in isolation. When there are multiple markets tied together through trade, optimal research may occur when demand is inelastic. Figure 2 provides a simple illustration of a multiple market situation. Abstracting from trade price differentials across markets, Figure 2 shows the demand and marginal revenue for the world in the short-run, with world production initially fixed at S1 and some country's production contribution at Q1. The price for this situation is p1. The unitary price elasticity of demand (-1) is at the point where the marginal revenue is zero (marginal revenue intersects the x or quantity axis). To the right of this point, where S1 occurs, marginal revenue is negative and the price elasticity of demand (in absolute value) is less than unitary. If, say as a result of the application of some research finding, the country in question increases production from Q1 to Q2, world supply will increase by the same amount from S1 to S2, price will decrease from p1 to p2, and the total revenue for the country in question will decline by the amount of the rectangle enclosing the horizontal arrow (↔) and increase by the amount enclosing the vertical arrow (↕), and since the latter gain is larger than the former loss, the country's total revenue will increase. Thus, to reach the optimal level of research, it appears that expansion of production is needed with the optimal level of supply being at some point where demand is inelastic.[1]

Thus far, explicit supply and demand functions have been used to find the optimality conditions for allocation of advertising and research monies.. General-form supply and demand specifications, however, could also have been used for this analysis. For example, if equation (1) were specified as $q = q(p, A)$, first order condition (4a), $\partial\pi/\partial A = \partial pq/\partial A - 1 = 0$, would still hold (price fixed), and, multiplying this condition through by A, and multiplying and dividing by q, results in $A = pq\,(\partial q/\partial A)(A/q)$, which is the same as equation (5b) with $(\partial q/\partial A)(A/q) = \varepsilon_a$. In the analysis below, this general approach of specifying equations is taken.

Let q_1 and Q_1 be U.S. demand and supply for OJ; q_2 and Q_2 be rest of the world (ROW) demand and supply for OJ; p be the Brazil FOB price; c be a margin including the U.S. tariff plus other costs taking price up to the retail level, i.e., p+c is the U.S. retail price; m(R) be the margin between the FOB price and the grower price, i.e., p - m(R) is the grower price; and A and R are again advertising and research dollars. It is assumed that the U.S. is a net importer while the ROW is a net exporter, following McClain (1989). Excess demand and excess supply equations are defined to determine the import-export equilibrium. Brazil is the largest producer of OJ in the world accounting for over 50% of the world's production, while the U.S. accounts for about 30% of the world's total. Given the dominance of Brazil, its FOB price is used in the model. The U.S. FOB price differs from this price by the amount of the U.S. tariff and transportation costs.

[1] Figure 2 is not a proof, of course, that demand can be inelastic and a country can still have a research program consistent with maximizing net revenue. Specification of the country specific supply and demand equations underlying Figure 2 are needed to be more precise. This issue is further addressed in context of the world model for OJ developed in this section.

Price p is set such that excess demand in the U.S. (U.S. imports) is equal to excess supply in the ROW (ROW exports), given, c, m, A and R, i.e.,

(24) $q_1(p + c, A) - Q_1(p - m(R), R) = Q_2(p - m(R), R) - q_2(p)$.

The maximization problem for Florida and other U.S. growers is now

(25a) maximize $\pi = (p - m) Q_1(p - m, R) - (A+R)$,
subject to $X = A + R$,

where X is again the total money available for advertising and research. With Florida accounting for most of the production of OJ n the U.S., this problem will be discussed in context of Florida.

As before, the constraint is imposed by specifying $R = X\text{-}A$. That is, $m = m(X\text{-}A)$ and $R=X\text{-}A$ in the equation for π, i.e.,

(25b) $\max \pi = (p- m(X\text{-}A))Q_1(p - m(X\text{-}A), X\text{-}A) - X$.

The first order condition is

(26a) $\partial\pi/\partial A = Q_1\, \partial(p- m)/\partial A + (p- m)\, \partial Q_1(p - m, R)/\partial A = 0$,

or

(26b) $Q_1(\partial p/\partial A + \partial m/\partial R) + (p- m)(\partial Q_1/\partial p(\partial p/\partial A + \partial m/\partial R) - \partial Q_1/\partial R) = 0$,

or

(26c) $Q_1 (\partial p/\partial A + \partial m/\partial R) = - (p- m)(\partial Q_1/\partial p(\partial p/\partial A + \partial m/\partial R) - \partial Q_1/\partial R)$

or, after multiplying through by A, multiplying and dividing by (p-m), Q_1, and other variables, in select parts of the relationship,

(26d) $(p-m)Q_1((\partial p/\partial A)(A/p)(p/(p-m)) +(\partial m/\partial R)(R/m)((m/(p-m))(A/R)) =$
$- (p- m)Q_1((\partial Q_1/\partial p)(p/Q_1)((\partial p/\partial A)(A/p) + (\partial m/\partial R)(R/m)(m/p)(A/R)) - (\partial Q_1/\partial R)(R/Q_1)(A/R))$,

or

(26e) $(p-m)Q_1(\varepsilon_{pa} (p/(p-m)) + \varepsilon_{m,r}((m/(p-m))(A/R)) = -(p- m)Q_1(\varepsilon_{Q1,p}(\varepsilon_{pa} + \varepsilon_{m,r}(m/p)(A/R)) - \varepsilon_{Q1,r}(A/R))$,

where
$\varepsilon_{pa} = (\partial p/\partial A)(A/p)$,

$\varepsilon_{m,r} = (\partial m/\partial R)(R/m)$,
$\varepsilon_{Q1,p} = (\partial Q_1/\partial p)(p/Q_1)$,
$\varepsilon_{Q1,r} = (\partial Q_1/\partial R)(R/Q_1)$.

Further simplifying, the optimality condition is

(26f) $\varepsilon_{pa}(p/(p-m)) + \varepsilon_{m,r}((m/(p-m))(A/R) = -\varepsilon_{Q1,p}(\varepsilon_{pa} + \varepsilon_{m,r}(m/p)(A/R)) + \varepsilon_{Q1,r}(A/R))$,

or

(26g) $A/R = (\varepsilon_{pa}(p/(p-m)) + \varepsilon_{Q1,p}\,\varepsilon_{pa}) / (\varepsilon_{Q1,r} - \varepsilon_{m,r}((m/(p-m)) - \varepsilon_{Q1,p}\,\varepsilon_{m,r}(m/p))$.

In equation (26g), the elasticity ε_{pa} depends on the world excess demand and supply situation. Total differential of the excess demand and supply relationship, equation (24),under the restriction that R=X-A, yields

(27a) $(\partial q_1/\partial p)dp + (\partial q_1/\partial A)dA - (\partial Q_1/\partial p)(dp + (\partial m/\partial R)dA) + (\partial Q_1/\partial R)dA =$
$(\partial Q_2/\partial p)(dp + (\partial m/\partial R)dA) - (\partial Q_2/\partial R)dA - (\partial q_2/\partial p)dp$,

or

(27b) $dp = (-(\partial q_1/\partial A)dA + (\partial Q_1/\partial p + \partial Q_2/\partial p)(\partial m/\partial R)dA -$
$(\partial Q_1/\partial R + \partial Q_2/\partial R)dA) / (\partial q_1/\partial p + \partial q_2/\partial p - \partial Q_1/\partial p - \partial Q_2/\partial p)$,

or

(27c) $dp/dA = (-(\partial q_1/\partial A) + (\partial Q_1/\partial p + \partial Q_2/\partial p)(\partial m/\partial R) - (\partial Q_1/\partial R + \partial Q_2/\partial R)) /$
$(\partial q_1/\partial p + \partial q_2/\partial p - \partial Q_1/\partial p - \partial Q_2/\partial p)$.

Multiplying both side of result (27c) by A, and further multiplying and dividing by selected terms, results in

(27d) $(dp/dA)(A/p) = (-((\partial q_1/\partial A)(A/q_1)) + ((\partial Q_1/\partial p)(p/Q_1)(Q_1/q_1) + (\partial Q_2/\partial p)(p/Q_2)(Q_2/q_1))(\partial m/\partial R)(R/m)(m/p)(A/R) - ((\partial Q_1/\partial R)(R/Q_1)(Q_1/q_1)(A/R) + (\partial Q_2/\partial R)(R/Q_2)(Q_2/q_1)(A/R)) /$
$((\partial q_1/\partial p)(p/q_1) + (\partial q_2/\partial p)(p/q_2)(q_2/q_1) - (\partial Q_1/\partial p)(p/Q_1)(Q_1/q_1) - (\partial Q_2/\partial p0(p/Q_2)(Q_2/q_1))$.

or

(27e) $\varepsilon_{pa} = (\varepsilon_{q1a} - (\varepsilon_{Q1p}(Q_1/q_1) + \varepsilon_{Q2p}(Q_2/q_1))\,\varepsilon_{m,r}(m/p)(A/R) + \varepsilon_{Q1r}(Q_1/q_1)(A/R) + \varepsilon_{Q2r}(Q_2/q_1)(A/R)) / (\varepsilon_{Q1p}(Q_1/q_1) + \varepsilon_{Q2p}(Q_2/q_1 - \varepsilon_{q1p} - \varepsilon_{q2p}(q_2/q_1))$,

where
$\varepsilon_{Q2,p} = (\partial Q_1/ \partial p)(p/Q_1)$,
$\varepsilon_{Q2,r}=(\partial Q_2/ \partial R)(R/Q_2)$,
$\varepsilon q_{q1,p} = (\partial q_1/ \partial p)(p/q_1)$,
$\varepsilon_{Qq2,p} = (\partial q_2/ \partial p)(p/q_2)$.

Thus, the optimality condition is equation (26g) with ε_{pa} defined as in (27e). Re-writing equation (26g), factoring out ε_{pa}, results in

(28a) $A/R = \varepsilon_{pa}((p/(p-m)) + \varepsilon_{Q1,p}) / (\varepsilon_{Q1,r} - \varepsilon_{m,r}((m/(p-m)) - \varepsilon_{Q1,p}\, \varepsilon_{m,r}(m/p))$,

or, rearranging,

(28b) $(A/R)(\varepsilon_{Q1,r} - \varepsilon_{m,r}((m/(p-m)) - \varepsilon_{Q1,p}\, \varepsilon_{m,r}(m/p)) = \varepsilon_{pa}((p/(p-m)) + \varepsilon_{Q1,p})$,

or, substituting the right-hand side of equation (27e) into equation (28b) and rearranging, results in the optimality condition

(28c) $A/R = [\, \varepsilon_{q1a}\, ((p/(p-m)) + \varepsilon_{Q1,p})] /$
$[(\varepsilon_{Q1,r} - \varepsilon_{m,r}((m/(p-m)) - \varepsilon_{Q1,p}\, \varepsilon_{m,r}(m/p))(\varepsilon_{Q1p}(Q_1/q_1) + \varepsilon_{Q2p}\,(Q_2/q_1) - \varepsilon_{q1p} - \varepsilon_{q2p}(q_2/q_1)))$
$+((\varepsilon_{Q1p}\,(Q_1/q_1) + \varepsilon_{Q2p}\,(Q_2/q_1))\, \varepsilon_{m,r}\,(m/p) - \varepsilon_{Q1r}(Q_1/q_1) - \varepsilon_{Q2r}(Q_2/q_1))((p/(p-m)) + \varepsilon_{Q1,p})]$.

As found above, equation (23g), result (28c) shows that the A/R ratio increases proportionately with the elasticity of demand with resepct to advertising (ε_{q1a}). The relationship with research, however, is more involved now. The first term in parentheses in the denominator ($\varepsilon_{Q1,r} - \varepsilon_{m,r}((m/(p-m)) - \varepsilon_{Q1,p}\, \varepsilon_{m,r}(m/p)$) is positive assuming the research elasticity is positive ($\varepsilon_{Q1,r} > 0$) and the margin elasticity is negative($\varepsilon_{m,r} < 0$; an increase in research reduces the margin). Given the supply and demand slopes are positive and negative, respectively, the next term ($\varepsilon_{Q1p}(Q_1/q_1) + \varepsilon_{Q2p}\,(Q_2/q_1) - \varepsilon_{q1p} - \varepsilon_{q2p}(q_2/q_1)$) is postive, and with this term multiplied by the first term first, we find that research that shifts supply and reduces the margin has a direct impact favoring research or reducing the A/R ratio. Research also has an indirect impact through its impact on price, equation (27e). Research increases supply which results in a price reduction and the remaining terms in the denominator of equation (28c) capture this effect which does not favor research.

Changes to reach the optimal production level can be determined from totally differentiating the U.S. production equation, $Q_1(p - m(R), R)$, i.e.,

(29) $dQ_1 = (\partial Q_1/\partial p)(dp + (\partial m/\partial R)dA) - (\partial Q_1/\partial R)dA$.

In equation (29), dp is set according to equation (27b).

Numerical Analysis

Using equation (28c), the optimal ratio of advertising to research expenditures was calculated for various assumed values of the right-hand side elasticities. Price (p), the margin (m) and quantities supplied (Q_1, Q_2) and demanded (q_1, q_2) are set at arbitrary values as an approximation (Table 1).[2] It is also assumed that the elasticities of supply with respect to price, and the elasticities of supply with respect to research are the same in the U.S. and the ROW.

The various optimal ratios of advertising to research expenditures calculated tended to favor advertising at current estimated elasticities of demand for the U.S. and the ROW. These results are related to the point discussed above that when demand is inelastic, additional research that increases production results in less revenue. The demand relationships considered for the U.S. and the ROW were both inelastic (Spreen et al; Brown et al), but given the structure of the world model, the optimal research focusing on increasing production was positive but small. Research focusing on the price margins was more effective since associated margin declines result in increases in the grower price and the quantity supplied according to the grower supply relationship (the grower supply curve remains fixed but more is supplied at a higher grower price).

The results are dramatically different at higher elasticities of demand with respect to price. If U.S. and ROW production of OJ declined, for example, by 75% due to greening, the price of OJ would be expected to increase sharply. At such higher prices and lower quantities, the U.S. and ROW elasticities of demand with respect to price would be expected to be much higher and in the elastic range, at least in the case of linear demands with respect to price. The last three rows of Table 2 provide scenarios assuming quantities are reduced by 75% and the FOB price increases to $4.00/ps. In this case, research dominates advertising in terms of generating revenue.

To examine the sensitivity of the analysis to the price, margin and quantity assumptions, the optimal levels of advertising and research were also calculated for all of the scenarios in Table 2, with the values for the price (p), margin (m) and quantities supplied (Q_1, Q_2) and demanded (q_1, q_2) increased and decreased by 50%. The 50% increase in these values resulted in a 12.7% decline in the A/R ratio on average across the scenarios, while the 50% increase resulted in an 18.8% increase in the ratio.

The scenarios considered here do not address a more speculative but important research possibility, the development of new products and fruit varieties with new demands. Strong demands for new products may result in relatively high prices, volume sales, and high rates of return for the research.

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2 Except for the margin (m), these variables depend on the levels of advertising (A) and research (R).

Summary

This study shows how the mix of advertising and research that maximizes net revenue for a commodity can be approximated for given values of market prices, margins and quantities, along with price elasticities of supply and demand, and the elasticity of demand with respect to advertising and the elasticity of supply and the price margin with respect to research.

In the simple supply and demand model, the revenue maximizing advertising-research ratio A/R increases proportionately with the elasticity of demand with respect to advertising, and decreases with the elasticity of supply with respect to research. It was also found that in this model that demand must be elastic for a positive R. For revenue maximization in the case where demand is linear with respect to price, or for specifications where the price elasticity of demand varies from the elastic to inelastic ranges, we find that when the industry is operating at a point where demand is elastic, research would be used at most to increase production to the point where demand has a unitary price elasticity; if production occurred at a point where demand were inelastic, no research would occur and quantity sold would be reduced until marginal revenue were zero or demand again has a unitary price elasticity.

Although there is not an exact carryover of the results of the simple model to our world model, the basic driving foruces in the simple model are found in the world model. It was found that for a single country there can be postive research although demand is inelastic. The overall impact of research is more involved through its effect on the world price.

The numerical analysis for the Florida OJ industry illustrates how the advertising-research ratio varies with alternative assumptions on the elastcities of the model, as well as prices, the margin and quantities. At current estimated values for these parameters, advertising tends to be favored over research, but for major shifts in supply due say to greening and canker, research expenditures exceed those for advertising.

Table 1. Price and Quantity Assumptions

p	$/ps	1.50
m	$/ps	0.60
p-m	$/ps	0.90
p/(p-m)	$/ps	1.67
m/(p-m)	$/ps	0.67
m/p	$/ps	0.40
Q1	mil. ps	950
Q2	mil. ps	2250
q1	mil. ps	1200
q2	mil. ps	2000

Table 2. Optimal Advertising-Research Ratios for Alternative Elasticities.

Elasticity of U.S. Demand wrt Advertising	Elasticity of Price Margin (FOB-Grower) wrt Researc h	Elasticities of Supplies(Q1 and Q2) wrt Research	Elasticities of Supplies(Q1 and Q2) wrt Price	Elasticity of U.S. Demand(q1) wrt Price	Elasticity of ROW Demand(q2) wrt Price	Optimal Advertising /Research Expenditures
Eq1a	Emr	EQr	EQp	Eq1p	Eq2p	A/R
0.050	-0.500	0.000	0.500	-0.229	-0.300	**0.343**
0.150	-0.500	0.000	0.500	-0.229	-0.300	**1.029**
0.300	-0.500	0.000	0.500	-0.229	-0.300	**2.058**
0.150	-0.250	0.000	0.500	-0.229	-0.300	**2.058**
0.150	-0.500	0.000	0.500	-0.229	-0.300	**1.029**
0.150	-0.750	0.000	0.500	-0.229	-0.300	**0.686**
0.150	-0.500	0.010	0.500	-0.229	-0.300	**1.166**
0.150	-0.500	0.020	0.500	-0.229	-0.300	**1.346**
0.150	-0.500	0.030	0.500	-0.229	-0.300	**1.590**
0.150	-0.500	0.020	0.250	-0.229	-0.300	**1.402**
0.150	-0.500	0.020	0.500	-0.229	-0.300	**1.346**
0.150	-0.500	0.020	1.000	-0.229	-0.300	**1.272**
0.050	-0.750	0.020	1.000	-0.229	-0.300	**0.262**
0.300	-0.500	0.500	0.500	-1.500	-1.500	**0.358**
0.300	-0.500	0.500	0.500	-3.000	-3.000	**0.129**
0.300	-0.500	0.500	0.500	-4.500	-4.500	**0.078**

Figure 1

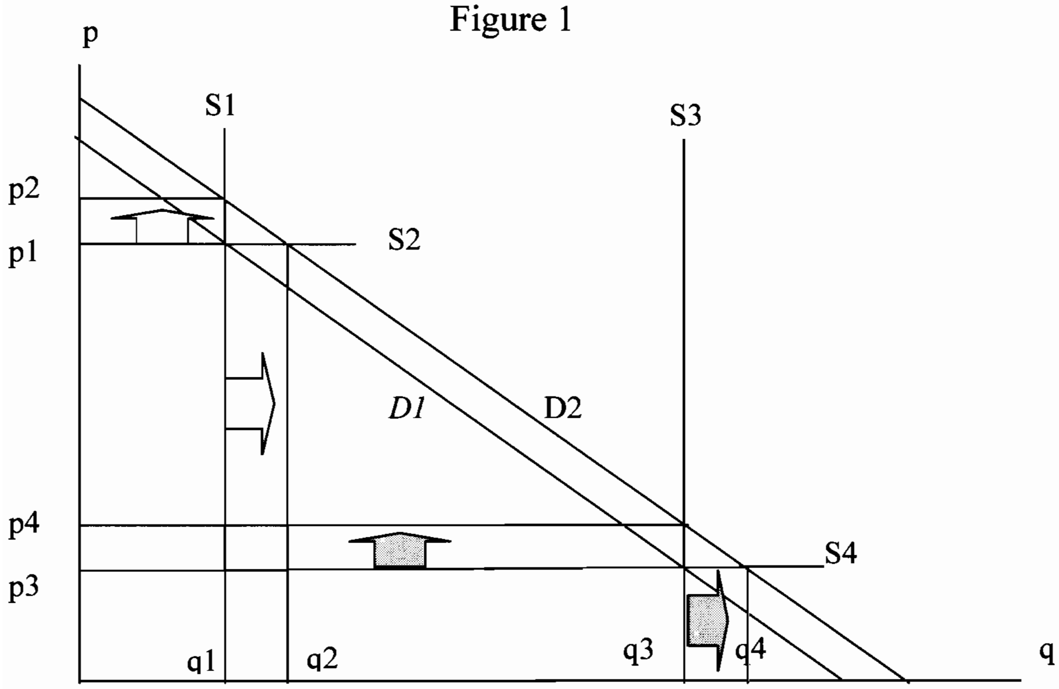
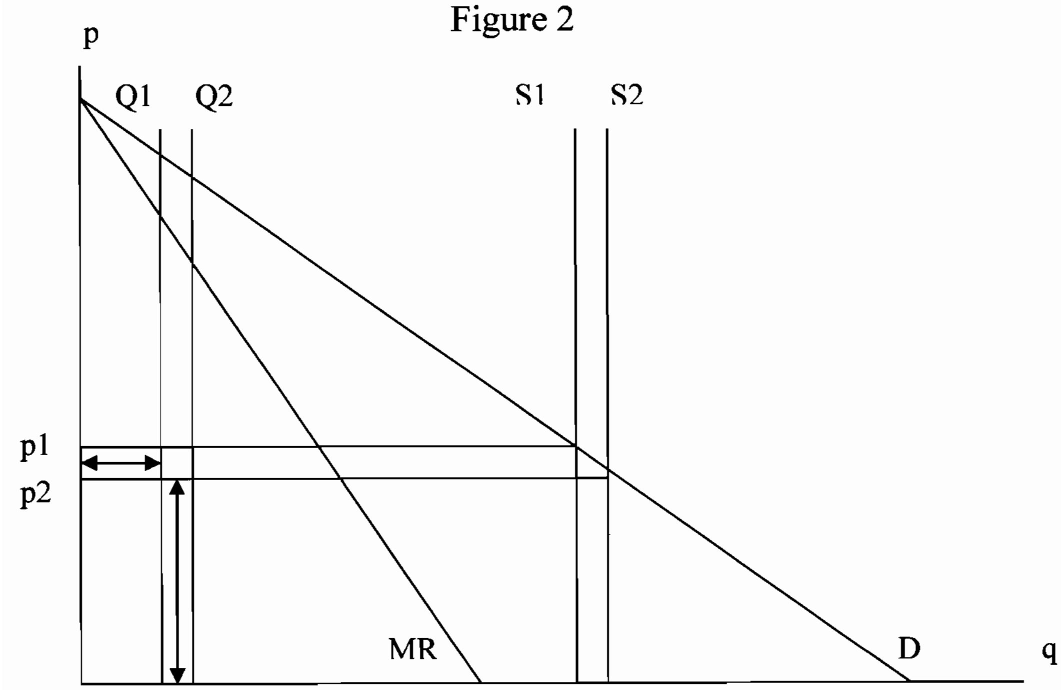


Figure 2



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